Nonequilibrium dynamics of a two-channel Kondo system due to a quantum quench

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Recent experiments by Potok *et al.* have demonstrated a remarkable tunability between a single-channel Fermi-liquid fixed point and a two-channel non-Fermi-liquid fixed point. Motivated by this we study the nonequilibrium dynamics due to a sudden quench of the parameters of a Hamiltonian from a single-channel to a two-channel anisotropic Kondo system. We find a distinct difference between the long-time behavior of local quantities related to the impurity spin as compared to that of bulk quantities related to the total (conduction electrons plus impurity) spin of the system. In particular, the local impurity spin and the local spin susceptibility are found to equilibrate but in a very slow power-law fashion which is peculiar to the non-Fermi-liquid properties of the Hamiltonian. In contrast, we find a lack of equilibration in the two-particle expectation values related to the total spin of the system.

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I. INTRODUCTION

The behavior of a local spin coupled to one or more independent channels of conduction electrons is a classic problem in condensed-matter physics. It is well known that when the local spin has a size $S=1/2$ and is coupled to only a single channel of conduction electrons, the spin is completely screened and the many-particle system behaves as a Fermi liquid.[1] In contrast, when the local spin is coupled to two or more screening channels, one has dramatically different behavior where the spin is overscreened, and the system exhibits non-Fermi-liquid properties.[2] Due to the success in realizing nanostructures consisting of a spin coupled to one or more reservoirs, there has been a resurgence of interest in these classic systems. The primary focus now is on understanding their nonequilibrium properties, such as the effect of current flow[3] and their nonequilibrium time evolution.[4]

In this paper we will study nonequilibrium dynamics of a two-channel Kondo system. We are motivated by recent experiments by Potok *et al.*[5] where by tuning external gate voltages a local spin could be effectively coupled to a single screening channel or to two independent screening channels. Thus as a function of gate voltage, both single-channel Fermi-liquid physics as well as two channel non-Fermi-liquid physics was demonstrated on the same device. We will study what happens when this external gate voltage controlling the fine tuning is changed rapidly in time from an initial value corresponding to a single-channel Kondo (1CK) system to a final value corresponding to a two-channel Kondo (2CK) system, thus inducing nonequilibrium dynamics. The time evolution of both single-particle expectation values, as well as two-particle expectation values that exhibit non-Fermi-liquid behavior in equilibrium will be studied.

Our system consists of two chiral noninteracting fermions that constitute the two channels that interact with the local spin $S=1/2$. We will employ the Emery-Kivelson mapping onto an interacting resonant-level model.[6] In the past any dynamics using this mapping, besides addressing other physical situations, was studied only at the noninteracting Toulouse point.[7] In this paper, in order to capture any nontrivial dynamics of the total spin of the system, we will have to move away from the Toulouse point. The Hamiltonian H is

$$H = iv_F\sum_{\alpha,i=1,2}\int_{-\infty}^{\infty}dx\psi^{\dagger}_{i\alpha}\frac{\partial}{\partial x}\psi_{i\alpha}(x)+\frac{h_1}{2}\tau^z + \frac{h_2}{2}\sum_{i=1,2}\int dx(\psi^{\dagger}_{i\uparrow}\psi_{i\uparrow}-\psi^{\dagger}_{i\downarrow}\psi_{i\downarrow}) + \frac{1}{2}\sum_{\substack{\alpha\beta,i=1,2\\ \lambda=x,y,z}}J^{\lambda}_i(t)\tau^{\lambda}\psi^{\dagger}_{i\alpha}(0)\sigma^{\lambda}_{\alpha\beta}\psi_{i\beta}(0). \tag{1}$$

Above, i labels channel while α,β labels spin index. $\vec{\tau},\vec{\sigma}$ are Pauli matrices and $\frac{1}{2}\psi^{\dagger}_{i\alpha}\vec{\sigma}_{\alpha\beta}\psi_{i\beta}$ is the spin-density operator for the electrons in the ith channel, while $\vec{S}=\vec{\tau}/2$ is the impurity spin operator. The coupling to the leads $J^{\lambda}_i(t)$ are time dependent (in an experiment these may be tuned by external gate voltages).[5] When $h_1=h_2$, a uniform magnetic field couples equally to both the impurity spin as well as the spins in the leads. When $h_2=0,\ h_1\neq 0$, the magnetic field couples only to the impurity spin. We will assume anisotropic couplings $J^x_i(t)=J^y_i(t)=J^{\perp}_i(t)\neq J^z_i(t)$. It is convenient to define, $\bar{J}_{z,\perp}=(J^{z,\perp}_1+J^{z,\perp}_2)/2,\ \ \delta J_{z,\perp}=J^{z,\perp}_1-J^{z,\perp}_2$, where $\delta J_{z,\perp}=0$ at the 2CK fixed point.

We briefly review the steps involved in mapping the above model onto an interacting resonant-level model.[6] One defines the canonically conjugate variables $[\phi_{i\alpha}(x),\Pi_{j\beta}(y)]=i\delta_{ij}\delta_{\alpha\beta}\delta(x-y)$ in terms of which the fermions are written as $\psi_{i\alpha}(x)=\exp[-i\Phi_{i\alpha}(x)]\eta_{i\alpha}/\sqrt{2\pi\alpha}$, where $\Phi_{i\alpha}(x)=\sqrt{\pi}[\int_{-\infty}^{x}dx'\Pi_{i\alpha}(x')+\phi_{i\alpha}(x)]$. This ensures that the same species of fermions anticommute with each other. $\eta_{i\alpha}$ are the Klein factors that are necessary to ensure anticommutation between different species of fermions. We choose[8] $\eta_{i\alpha}=\exp(i\theta^K_{i\alpha})$ where, $\theta^K_{1\downarrow}=0;\ \theta^K_{1\uparrow}=\pi N_{1\downarrow};\ \theta^K_{2\downarrow}=\pi(N_{1\downarrow}+N_{1\uparrow});\ \theta^K_{2\uparrow}=\pi(N_{1\downarrow}+N_{1\uparrow}+N_{2\downarrow}),\ \ N_{i\alpha}$ being the total number of $i\alpha$ fermions. Defining $\chi_{i\alpha}=\Phi_{i\alpha}-\theta^K_{i\alpha}$, one changes variables to $2\chi_c=\chi_{1\uparrow}+\chi_{1\downarrow}+\chi_{2\uparrow}+\chi_{2\downarrow}$, $2\chi_s=\chi_{1\uparrow}-\chi_{1\downarrow}+\chi_{2\uparrow}-\chi_{2\downarrow}$, $2\chi_f=\chi_{1\uparrow}+\chi_{1\downarrow}-\chi_{2\uparrow}-\chi_{2\downarrow}$, $2\chi_{sf}=\chi_{1\uparrow}-\chi_{1\downarrow}-\chi_{2\uparrow}+\chi_{2\downarrow}$. Next one performs the unitary transformation $H\rightarrow U^{\dagger}HU$, where $U=\exp[-iS_z\chi_s(0)]$, followed by a refermionization of the Hamiltonian into the fermionic fields $d^\dagger=-iS^+$; $d=iS^-$ (so that $d^\dagger d-\frac{1}{2}=S^z$) and $\psi_{\nu=c,s,f,sf}(x)=e^{i\pi d^\dagger d}e^{-i\chi_\nu(x)}/\sqrt{2\pi\alpha}$. In what follows we will assume that $J^z_{1,2},J_{1\perp}$ are time independent and $\delta J^z=0$. We set $J_{1\perp}=J_\perp$, and the only time dependence will be in $J_{2\perp}(t)$. With this we obtain

$$U^\dagger HU = iv_F\sum_{\nu=c,s,f,sf}\int_{-\infty}^{\infty}dx\left(\psi_\nu^\dagger\frac{\partial\psi_\nu}{\partial x}\right) + h_2\int_{-\infty}^{\infty}dx\psi_s^\dagger(x)\psi_s(x) + (h_1-h_2)\left(d^\dagger d-\frac{1}{2}\right) + 2(\bar{J}^z-\pi v_F)\left(d^\dagger d-\frac{1}{2}\right):\psi_s^\dagger(0)\psi_s(0): + \frac{J_\perp}{\sqrt{2\pi\alpha}}[d^\dagger\psi_{sf}(0)+\psi_{sf}^\dagger(0)d] + \frac{J_{2\perp}(t)}{\sqrt{2\pi\alpha}}[d^\dagger\psi_{sf}^\dagger(0)+\psi_{sf}(0)d]. \quad (2)$$

We will assume that the time dependence of $J_{2\perp}(t)$ is that of a quench, $J_{2\perp}(t)=J_\perp\theta(t)$. Thus for $t<0$ we have a 1CK system that is described by an interacting resonant-level model. Whereas for $t>0$, the Hamiltonian is that of a 2CK system where the coupling of the resonant level to the reservoir of ψ_{sf} fermions is via $\frac{J_\perp}{\sqrt{2\pi\alpha}}[(d^\dagger-d)\psi_{sf}(0)+\text{H.c.}]$. Thus in the 2CK model effectively only half of the resonant level corresponding to the Majorana fermion $a=-i(d^\dagger-d)/\sqrt{2}$ couples to the conduction electrons, while the other half $b=(d^\dagger+d)/\sqrt{2}$, does not couple. As was pointed out in Ref. 6 all non-Fermi-liquid behavior stems from this peculiarity of the resonant level, and as we shall see is also responsible for interesting behavior in the dynamics.

To see this note that immediately after the quench we have a highly nonequilibrium system, where any local degrees of freedom can relax to the ground state only via their coupling to the reservoirs. In the 2CK model since only half the local degrees of freedom are coupled, local quantities relax very slowly, as we shall show in a power-law manner. Moreover we find that two-particle expectation values related to the total (bulk plus local) spin of the system do not relax to their equilibrium values.

II. TIME EVOLUTION OF LOCAL QUANTITIES

We will first consider the case when the external magnetic field couples only to the local spin so that $h_1=h$ and $h_2=0$. We will study the time evolution of the local magnetization, and the local spin susceptibility, the latter in the limit $h\to 0$. To capture the non-Fermi-liquid behavior of the local susceptibility in an equilibrium 2CK system, it suffices to be at the noninteracting Toulouse point $\bar{J}^z=\pi v_F$. Therefore the nonequilibrium dynamics of the local quantities will also be studied at the Toulouse point. Later while studying the dynamics of the total spin, we will have to move away from the Toulouse point so as to capture non-Fermi-liquid physics.[9,10] We define the following Green's function for the local fermion (spin),

$$\hat{G}^R(t,t') = -i\theta(t-t')\left\langle\left\{\begin{pmatrix} d(t)\\ d^\dagger(t)\end{pmatrix},(d^\dagger(t')\quad d(t'))\right\}\right\rangle, \quad (3)$$

$$\hat{G}^K(t,t') = -i\left\langle\left[\begin{pmatrix} d(t)\\ d^\dagger(t)\end{pmatrix},(d^\dagger(t')\quad d(t'))\right]\right\rangle. \quad (4)$$

Denoting the individual elements of the above matrices as $\hat{G}=\begin{pmatrix} G_{d,d^\dagger} & G_{d,d}\\ G_{d^\dagger,d^\dagger} & G_{d^\dagger,d}\end{pmatrix}$, $G^{R,K}$ obey the equation of motion,

$$\left[i\partial_t - h\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} - \hat{\Sigma}^R\circ\right]\hat{G}^R = 1, \quad (5)$$

$$\hat{G}^K = \hat{G}^R\circ\hat{\Sigma}^K\circ\hat{G}^A, \quad (6)$$

where $\circ$ denotes convolution in time, $G^A(t,t')=[G^R(t',t)]^*$, and $\Sigma^{R,K}$ are the self-energies due to coupling to the leads. Defining $\Gamma_\perp=\frac{J_\perp^2}{\pi\alpha v_F}$, a time dependence of the form $J_{2\perp}(t)=J_\perp\theta(t)$ implies

$$\hat{\Sigma}^R(t,t') = \frac{-i\Gamma_\perp}{4}\delta(t-t')\begin{pmatrix}1+\theta^2(t) & -2\theta(t)\\ -2\theta(t) & 1+\theta^2(t)\end{pmatrix}, \quad (7)$$

$$\hat{\Sigma}^K(t,t') = -\frac{\Gamma_\perp}{2}P\left[\frac{T}{\sinh \pi T(t-t')}\right]\times\begin{pmatrix}1+\theta(t)\theta(t') & -(\theta(t')+\theta(t))\\ -(\theta(t')+\theta(t)) & 1+\theta(t)\theta(t')\end{pmatrix}, \quad (8)$$

where T is the temperature of the conduction electrons.

The solutions to Eq. (5) depend on whether the time arguments in $G^R(t,t')$ are before or after the quench. When both times are before the quench,

$$\hat{G}^R(t<0,t'<0) = -i\theta(t-t')e^{-\Gamma_\perp(t-t')/4}\begin{pmatrix}e^{-ih(t-t')} & 0\\ 0 & e^{ih(t-t')}\end{pmatrix}. \quad (9)$$

When both times are after the quench we get

$$\hat{G}^R(t>0,t'>0) = -i\theta(t-t')e^{-\Gamma_\perp(t-t')/2}\times\begin{pmatrix}A_1(t,t') & B_1(t,t')\\ B_1(t,t') & [A_1(t,t')]^*\end{pmatrix}, \quad (10)$$

where $A_1(t,t')=\cosh[\sqrt{\frac{\Gamma_\perp^2}{4}-h^2}(t-t')] - \frac{ih}{\sqrt{\frac{\Gamma_\perp^2}{4}-h^2}}\sinh[\sqrt{\frac{\Gamma_\perp^2}{4}-h^2}(t-t')]$ and $B_1(t,t') = \frac{\Gamma_\perp/2}{\sqrt{\frac{\Gamma_\perp^2}{4}-h^2}}\sinh[\sqrt{\frac{\Gamma_\perp^2}{4}-h^2}(t-t')]$. When one of the times is after the quench and the other before

$$\hat{G}^R(t>0,t'<0) = -i\theta(t-t')e^{-\Gamma_\perp t/2+\Gamma_\perp t'/4}\times\begin{pmatrix}A_2(t,t') & B_2(t,t')\\ B_2(t,t') & [A_2(t,t')]^*\end{pmatrix}, \quad (11)$$

where $A_2(t,t')=\cosh\sqrt{(\frac{\Gamma_\perp t}{2})^2-h^2(t-t')^2}$

$-\frac{ih(t-t')}{\sqrt{(\frac{\Gamma_\perp t}{2})^2-h^2(t-t')^2}}\sinh\sqrt{(\frac{\Gamma_\perp t}{2})^2-h^2(t-t')^2}$ and $B_2(t,t') = \frac{\Gamma_\perp t/2}{\sqrt{(\frac{\Gamma_\perp t}{2})^2-h^2(t-t')^2}}\sinh\sqrt{(\frac{\Gamma_\perp t}{2})^2-h^2(t-t')^2}$.

We first discuss the behavior of the magnetization $S^z=\frac{-i}{2}G^K_{d,d^\dagger}(t,t)$ for a time after the quench (hence, $t>0$) and when the conduction electrons are at a temperature $T=0$. Substituting Eqs. (8), (10), and (11) in Eq. (6), in the limit of $h \ll \Gamma_\perp$ and long times $t \gg 1/h$ we find

$$S^z(t) - S^z_{2\mathrm{CK},eq} \sim e^{-h^2 t/\Gamma_\perp}\left[\frac{1}{\pi h t} + \mathcal{O}\left(\frac{1}{h^2t^2}, \frac{1}{\Gamma_\perp t}\right)\right], \quad (12)$$

where $S^z_{2\mathrm{CK},eq}=-\frac{h}{2\pi\sqrt{\Gamma_\perp^2-4h^2}}\ln[\frac{\Gamma_\perp^2-2h^2+\Gamma_\perp\sqrt{\Gamma_\perp^2-4h^2}}{\Gamma_\perp^2-2h^2-\Gamma_\perp\sqrt{\Gamma_\perp^2-4h^2}}]$ is the local magnetization in the ground state of the 2CK Hamiltonian. Thus the local magnetization does equilibrate but at a slow rate of $h^2/\Gamma_\perp$ associated with the b fermion. In contrast, for a reverse quench $J_{2\perp}(t)=J_\perp\theta(-t)$ where the time evolution is governed by a 1CK model for which the a and b fermions are equally coupled to the reservoirs, we have checked that S_z equilibrates at the much faster rate of $\Gamma_\perp/4$.

We will now study the time evolution of the local longitudinal spin response function $\chi^R_{loc}(t,t')=-i\theta(t-t')\langle\{d^\dagger(t)d(t), d^\dagger(t')d(t')\}\rangle$ which we rewrite as

$$\chi^R_{loc}(t,t') = \frac{-i}{2}[G^R_{d,d^\dagger}(t,t')G^K_{d,d^\dagger}(t',t) + G^K_{d,d^\dagger}(t,t')G^A_{d,d^\dagger}(t',t) - G^R_{d^\dagger,d^\dagger}(t,t')G^K_{d,d}(t',t) - G^K_{d^\dagger,d^\dagger}(t,t')G^A_{d,d}(t',t)]. \quad (13)$$

It is useful to define the nonequilibrium static susceptibility at time T_m, $\chi_{S,loc}(T_m)=\int_0^\infty d\tau\chi^R_{loc}(T_m+\frac{\tau}{2}, T_m-\frac{\tau}{2})$. For $h=0$, and for very low temperatures $T \ll \Gamma_\perp$ of the conduction electrons, we find the following behavior for the static susceptibility at times $T_m \gg 1/\Gamma_\perp$,

$$\begin{aligned}\chi_{S,loc}(T_m) - \chi^{eq}_{S,loc,2\mathrm{CK}} &\sim \frac{1}{\pi\Gamma_\perp}\ln\left(\frac{1}{2TT_m}\right) + \frac{1}{\pi\Gamma_\perp}\mathcal{O}\left(\frac{1}{\Gamma_\perp T_m}\right) \quad \forall \quad TT_m \ll 1\\ &\sim \frac{1}{\pi\Gamma_\perp}\left(\frac{1}{2TT_m}\right) \quad \forall \quad TT_m \gg 1,\end{aligned} \quad (14)$$

where[6,10] $\chi^{eq}_{S,loc,2\mathrm{CK}}=\frac{1}{\pi\Gamma_\perp}\ln\frac{T}{\Gamma_\perp}$ is the equilibrium (non-Fermi-liquid) local susceptibility of the 2CK Kondo system. Thus we find that the logarithmic singularity associated with the 2CK system is cutoff by $\max(T, \frac{1}{T_m})$. Moreover at long times $TT_m \gg 1$, the local susceptibility equilibrates, but in a very slow power-law fashion which is determined by the temperature of the leads.

III. TIME EVOLUTION OF BULK PLUS LOCAL QUANTITIES

Let us consider the case where an external magnetic field couples to the total (conduction electrons plus local) spin of the system so that $h_1=h_2=h$. We will discuss

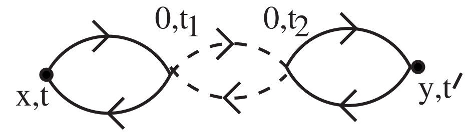

FIG. 1. Diagram for $\chi^R_{imp}(xt,yt')$. Dark line: propagator for the conduction electrons ψ_s. Dashed line: propagator for the local d fermion.

the time evolution of the response function of the total spin of the system when $h\to 0$. From Eq. (2), this may be formally defined as $\chi^R(x,t;yt') = -i\theta(t-t')\langle\{\psi^\dagger_s(x,t)\psi_s(x,t), \psi^\dagger_s(y,t')\psi_s(y,t')\}\rangle$. At the Toulouse point $\bar{J}^z=\pi v_F$, the local degrees of freedom do not couple to the bulk field ψ_s, so that the response function is independent of the local quench and is given by the Lindhard function,

$$\chi^R_0(q,\Omega) = \left(\frac{-L}{2\pi v_F}\right)\frac{qv_F}{qv_F-(\Omega+i\delta)}. \quad (15)$$

Thus the static spin susceptibility $\chi_{S0}(q,\Omega=0)=(\frac{-L}{2\pi v_F})$ and is independent of q.

To obtain non-Fermi-liquid behavior one has to move away from the Toulouse point,[9,10] which couples ψ_s to the local field, and also introduces nonequilibrium dynamics in χ^R. Defining, $\chi^R(q;t,t')=\int dx\int dy\cos q(x-y)\chi^R(x,t;yt')$, the leading correction in $(\bar{J}^z-\pi v_F)$ to χ^R (shown in Fig. 1) is

$$\begin{aligned}\chi^R_{imp}(q;t,t') = (\bar{J}^z-\pi v_F)^2\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{\infty}dt_2dxdy\chi^R_{loc}(t_1,t_2)\\ \cos[q(x-y)][G^R_{\psi^\dagger_s\psi_s}(x,0;t,t_1)G^K_{\psi^\dagger_s\psi_s}(0,x;t_1,t)\\ + G^K_{\psi^\dagger_s\psi_s}(x,0;t,t_1)G^A_{\psi^\dagger_s\psi_s}(0,x;t_1,t)]\\ \times[G^R_{\psi^\dagger_s\psi_s}(0,y;t_2,t')G^K_{\psi^\dagger_s\psi_s}(y,0;t',t_2)\\ + G^K_{\psi^\dagger_s\psi_s}(0,y;t_2,t')G^A_{\psi^\dagger_s\psi_s}(y,0;t',t_2)],\end{aligned} \quad (16)$$

where we have assumed that the interaction $(J^z-\pi v_F)$ has been switched on adiabatically slowly at long times in the past. The label χ_{imp} signifies that it is the correction to the bulk response function due to coupling to the local impurity, $G^{R,K}_{\psi^\dagger_s\psi_s}$ are the Green's function of the free ψ_s fermions, and χ^R_{loc} is defined in Eq. (13). Defining $t=T_m+\frac{\tau}{2}$; $t'=T_m-\frac{\tau}{2}$, the nonequilibrium static susceptibility $\chi_{S,imp}(q,T_m) = \int_0^\infty d\tau\chi^R_{imp}(q;T_m+\frac{\tau}{2},T_m-\frac{\tau}{2})$ is

$$\begin{aligned}\chi_{S,imp}(q,T_m) = \frac{1}{4}(\bar{J}^z-\pi v_F)^2\int dt_1\int dt_2\chi^R_{loc}(t_1,t_2)\\ \times\int\frac{d\epsilon}{2\pi}e^{-2iT_m\epsilon+i\epsilon(t_1+t_2)}\frac{1}{L^2}[\chi^R_0(q,\epsilon)\chi^R_0(q,-\epsilon)\\ + \chi^R_0(-q,\epsilon)\chi^R_0(-q,-\epsilon)],\end{aligned} \quad (17)$$

where χ^R_0 is given in Eq. (15). In equilibrium, i.e., when $J_2=J_\perp$ and is independent of time, χ^R_{loc} is independent of t_1+t_2. Thus the time integral over t_1+t_2 forces $\epsilon=0$ in Eq.

(17). With this one recovers the equilibrium result[9,10] $\chi_{S,imp}(q)=\frac{1}{4}(\frac{\bar{J}_z-\pi v_F}{2\pi v_F})^2\chi^{eq}_{S,loc,2CK}=\chi^{eq}_{S,imp,2CK}$. To study the evolution of the static susceptibility after the quench, it is convenient to change variables in Eq. (17) to $T'=\frac{t_1+t_2}{2}$, $\tau=t_1-t_2$. Defining $u=(\frac{\bar{J}_z-\pi v_F}{2\pi v_F})$ and performing the integration over ϵ we get,

$$\chi^R_{S,imp}(q,T_m)=\frac{u^2}{4}\int dT'\int d\tau\chi^R_{loc}\left(T'+\frac{\tau}{2},T'-\frac{\tau}{2}\right)\times[qv_F\sin(2qv_F|T'-T_m|)]. \tag{18}$$

We will present results for $qv_F \ll \Gamma_\perp$ and times $T_m \gg 1/\Gamma_\perp$ so that terms that fall off as $\frac{1}{\Gamma_\perp T_m}$ or faster will be dropped. Further, we will consider two cases: one where $q=0$ and the other when $qv_F \gg (T,\frac{1}{T_m})$.

For $q=0$, note that we should first perform the T' integral in Eq. (18), and then set $q=0$. This gives

$$\chi_{S,imp}(q=0)=\frac{1}{2}\chi^{eq}_{S,imp,2CK}+\frac{1}{2}\chi^{eq}_{S,imp,1CK}, \tag{19}$$

where $\chi^{eq}_{S,imp,1CK}=-(\frac{\bar{J}_z-\pi v_F}{2\pi v_F})^2\frac{1}{\pi\Gamma_\perp}$ is the static susceptibility in the 1CK ground state. For the case $qv_F \gg (T,\frac{1}{T_m})$, dropping terms of $\mathcal{O}(\frac{1}{2qv_FT_m})$, we find

$$\begin{aligned}\chi_{S,imp}(qv_F) &= \chi^{eq}_{S,imp,2CK}\left[1-\frac{1}{2}\cos(2qv_FT_m)\right] + \frac{1}{2}\chi^{eq}_{S,imp,1CK}\cos(2qv_FT_m) - \frac{u^2}{4\pi\Gamma_\perp}\text{cosineintegral}(4TT_m) \\ &- \frac{u^2}{8\pi\Gamma_\perp}\left[\frac{q^2v_F^2}{\Gamma_\perp^2}\ln\frac{\Gamma_\perp}{2T}+\left(\ln\frac{qv_F}{2T}\right)+\cdots\right]\times\cos(2qv_FT_m) - \frac{u^2}{20\pi\Gamma_\perp}\frac{q^2v_F^2}{(\Gamma_\perp^2+q^2v_F^2)}g(TT_m),\end{aligned} \tag{20}$$

where $g(x\ll 1)\sim 1+\mathcal{O}(x^2)$, $g(x\gg 1)\sim\frac{1}{x}$, and $\cdots$ represent terms that are small in comparison to $\ln(\frac{qv_F}{2T})$, $\ln(\frac{\Gamma_\perp}{2T})$.

Thus we find a marked difference between the susceptibility at long times after the quench and the susceptibility in equilibrium $\chi^{eq}_{S,imp,2CK}$. While $\chi^{eq}_{S,imp,2CK}$ is independent of wave vector, the out of equilibrium susceptibility is strongly dependent on q, and does not even reach a time-independent steady state, but instead oscillates at frequency qv_F [Eq. (20)]. For intermediate times $TT_m \ll 1$, performing a time averaging so that terms that oscillate at qv_F go to zero, we find

$$\bar{\chi}_{S,imp}\left(qv_F \gg \frac{1}{T_m}\gg T\right)=\frac{u^2}{4\pi\Gamma_\perp}\left[\ln\frac{1}{4\Gamma_\perp T_m}-\frac{q^2v_F^2}{5(\Gamma_\perp^2+q^2v_F^2)}\right]. \tag{21}$$

Thus for an intermediate time which is longer, the lower the temperature, the logarithmic divergences associated with the bulk susceptibility $\bar{\chi}_{S,imp}(qv_F)$ not only get cutoff by inverse time [a result similar to Eq. (14) for the local susceptibility], it also acquires some q-dependent corrections. In contrast, at long times $TT_m \gg 1$, Eq. (20) implies that the time-averaged susceptibility at large wave vectors $qv_F \gg T$ is, $\bar{\chi}_{S,imp}(qv_F\gg T\gg\frac{1}{T_m})=\chi^{eq}_{S,imp,2CK}+\mathcal{O}(\frac{1}{TT_m})$, and therefore equilibrates.

The $q=0$ static susceptibility [Eq. (19)] on the other hand is found to reach a time-independent steady state which is an equal mixture of the nonanalytic in temperature form of the 2CK ground state, and the analytic in temperature form of the 1CK ground state. This lack of equilibration in bulk properties is consistent with nonequilibrium time evolution in integrable models where the system retains memory of its initial state. For local quantities on the other hand [Eqs. (12) and (14)], at least at the Toulouse point, the rest of the system to which they are coupled acts as a reservoir causing them to equilibrate, but at very slow rates compared to a 1CK model.

In summary we have studied the nonequilibrium dynamics in a 2CK system due to a quantum quench. Our results highlight how the non-Fermi-liquid properties of the system, along with its integrability affect the time evolution of single-particle and two-particle expectation values. An interesting question concerns the observability of the nonequilibrium dynamics presented here. Experiments may be characterized by two kinds of effects that have not been taken into account in the present treatment. One is that the system could be "open," i.e., coupled to some other modes such as phonons, leading to an external dissipation rate γ_{diss} which will eventually cause the system to equilibrate. The second effect could be deviations from integrability arising, for example, due to a nonlinear dispersion for the conduction electrons. Studying the consequence of these effects is very interesting and beyond the scope of this paper. However, one may still be able to speculate on the effect of an external dissipation. In particular, a characteristic of the 2CK system is slow power-law dynamics. Thus we expect that for weak dissipation $\gamma_{diss} \ll \Gamma_\perp$, the system will equilibrate slowly as $\frac{1}{\gamma_{diss}T_m}$, (where T_m is the time after the quench) so that a nonequilibrium/transient state can still exist for long-enough time scales to be observable. The results of this paper are also relevant for Kondo systems in cold-atom gases where dissipative effects are weak.[11]

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